

Theoretical Predictions for Pionium Searches

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Abstract

Characteristic properties of pionium $A_{2\pi}$ and associated low energy s-wave cross sections $\sigma(\pi^0\pi^0 \rightarrow \pi^0\pi^0)$, $\sigma(\pi^+\pi^- \rightarrow \pi^0\pi^0)$ and $\sigma(\pi^0\pi^0 \rightarrow \pi^+\pi^-)$ are investigated with a coupled channels potential model. Some experimental results and conclusions are to be reconsidered.

Like other charged particle pairs the dimesonic $\pi^+\pi^-$ system is expected to form a Coulomb bound atomic system known as ponium $A_{2\pi}$. Ponium is realized by Coulomb attraction and their detailed properties are affected by the hadronic interaction. It has a finite lifetime due to strong coupling into the energetically open $\pi^0\pi^0$ channel and pion decays in general.

We are treating ponium as a low energy s-wave coupled channel resonance with a coupled channels potential model

$$f_i'' + k_i^2 f_i = \sum_{j=1}^2 m_i V_{ij} f_j, \quad i = 1(2) \text{ for } \pi^+\pi^-(\pi^0\pi^0). \quad (1)$$

with $M_{\pi\pi} = 2\sqrt{k_i^2 + m_i^2}$ and the interaction potential

$$V_{11} = V^{\pi^+\pi^-} = \frac{1}{3}V_0^2 + \frac{2}{3}V_0^0 - \frac{e^2}{r}, \quad (2)$$

$$V_{12} = V_{21} = \frac{\sqrt{2}}{3}(V_0^2 - V_0^0), \quad (3)$$

$$V_{22} = V^{\pi^0\pi^0} = \frac{2}{3}V_0^2 + \frac{1}{3}V_0^0. \quad (4)$$

The potential matrix is determined with results of quantum inversion of the $T = 0, 2$ experimental and theoretical hadronic phase shifts $\delta_0^T(M_{\pi\pi})$ in the elastic domain for $M_{\pi\pi} \leq 1$ GeV respectively [1–3]. Symmetrized mesonic wave functions and the isospin projectors

$$P(T=0) = \frac{1 - \tau.\tau}{3}, \quad P(T=2) = \frac{2 + \tau.\tau}{3}, \quad (5)$$

are used to construct the potential matrix (2–4). The input phase shift function and Froggatt data [1] are shown in Fig. 1 with the implication that Lohse data [2] give a qualitative similar picture. Gelfand–Levitan–Machenko inversion potentials for Froggatt and Lohse inputs as well as the potential matrix is shown in Fig. 2. We verified that the inversion potentials are reproducing for all energies the input phase functions within 0.1 degrees and scattering lengths in Table I. With numerical integration of (1) and asymptotic matching to Coulomb and Bessel functions we determine the normalization constants (below threshold for the closed $\pi^+\pi^-$ channel) and the hadronic S-matrix. The pure Coulomb potential supports in principle an infinite set $|\pi^+\pi^-, nS \rangle, n = 1, 2, \dots$ of bound states. We take for the $\pi^+\pi^-$

asymptotic wave function a finite superposition of states. In particular we study the pure ground state resonance, which we call the *pionium-proper* with $n=1$, and excited states with $n=2-5$. The elastic $\pi^0\pi^0$ cross sections with the pionium-proper resonance is shown in Fig. 3 and reaction cross sections in Fig. 4. They are determined with standard expressions from the S-matrix

$$\sigma(\pi^{00} \rightarrow \pi^{00}) = \frac{\pi}{k_2^2} |1 - S_{22}|^2 \quad (6)$$

and

$$\sigma(\pi^{00} \rightarrow \pi^{+-}) = \frac{\pi}{k_2^2} (1 - |S_{22}|^2), \quad (7)$$

$$\sigma(\pi^{+-} \rightarrow \pi^{00}) = \frac{\pi}{k_1^2} (1 - |S_{11}|^2). \quad (8)$$

The pionium-proper resonance in the $\pi^0\pi^0$ channel, Fig. 3 (top), has a FWHM = $14^F(20^L)$ eV equivalent to a lifetime $\tau = 4.7^F(3.3^L) \times 10^{-17}$ sec. Its binding energy is $E_B = 2.445^F(2.407^L)$ keV in contrast to $E_B = 1.858$ keV for the 1S Coulomb ground state. The pionium resonance cross sections are large (4 barns) and equal in magnitude for all resonances $n = 1 - 5$. Their resonance widths are small and they follow very well the rule Γ_1/n^2 eV, see Table I. Adjacent to the resonances we find in the continuum as prominent and important feature a large threshold reaction cross section peak for $\pi^+\pi^- \rightarrow \pi^0\pi^0$ which is shown in Fig. 4. Like the pionium resonances this peak has a large cross section (\sim barn) and a width of \sim MeV.

We are aware of the claim that pionium has been seen in high energy experiments [4] and continued interest exists by experimentalists and theoreticians [5,6]. However, on the basis of our presented numerical results and the experiment description we claim that pionium does not necessarily explain the data in experiment [4]. Rather, the experimental $\pi^+\pi^-$ surplus is an effect of π^+, π^- *initial state interactions* with the strong Coulomb field of target nuclei and the strong kinematically caused energy dependence of the $\pi^+\pi^- \rightarrow \pi^0\pi^0$ cross section. The target nucleus Coulomb field causes for the incoming $\pi^+\pi^-$ pair enough shift of the *relative kinetic energy* towards higher energy values that the rapidly falling transition cross section

yields a reduced transition into the $\pi^0\pi^0$ channel and thus the surplus seen in experiment [4]. Furthermore, we cannot confirm the used formulas in [4], especially the assumption of a small disturbance of the pure Coulomb atomic wave function by the hadronic potential. Their $A_{2\pi}$ lifetime is two orders of magnitude larger than our calculations predict and the lifetimes increase according to our calculations with $\tau_n = \tau_1 n^2$ and not proportional n^3 as quoted in [4].

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TABLES

TABLE I. $\pi\pi$ s-wave and $A_{2\pi}$ pionium ground and excited states.

Model	$a_0^0[\mu^{-1}]$	$a_0^2[\mu^{-1}]$	$E_B(n)[\text{keV}]$	τ_n [s]	$\Gamma_n(FWHM)$ [eV]
Froggatt	0.31	-0.059			
n=1			2.445	4.7×10^{-17}	14.0
Lohse	0.30	-0.025			
n=1			2.407	3.3×10^{-17}	20.3
n=2			0.602	1.3×10^{-16}	5.06
n=3			0.267	3.0×10^{-16}	2.25
n=4			0.150	5.3×10^{-16}	1.27
n=5			0.096	8.3×10^{-16}	0.81

FIGURES

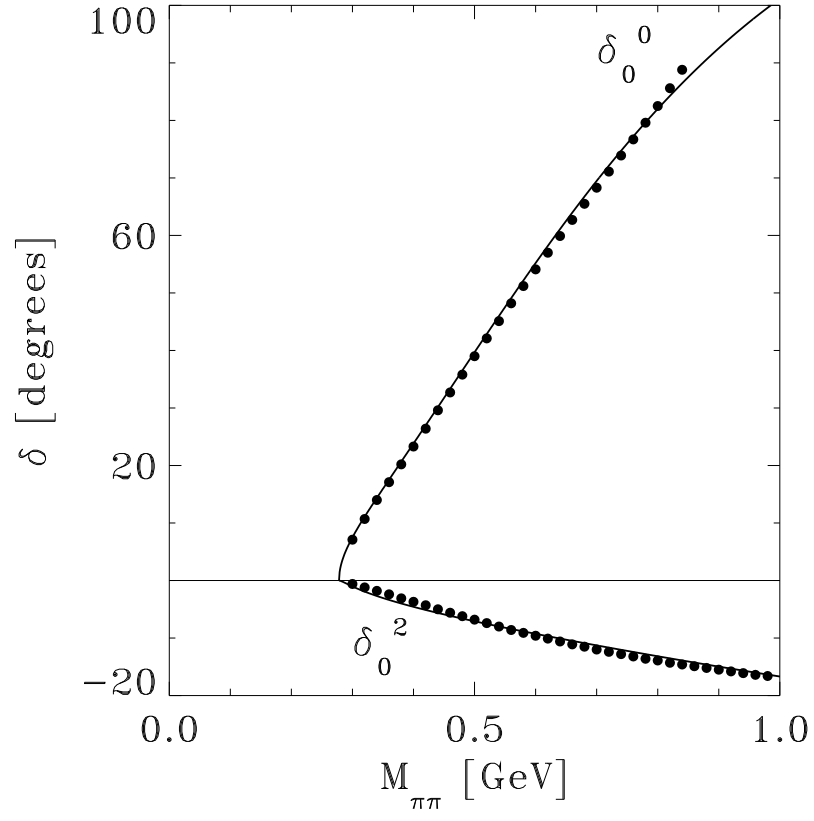


FIG. 1. Froggatt data [1] (dots) with input phase function (solid line).

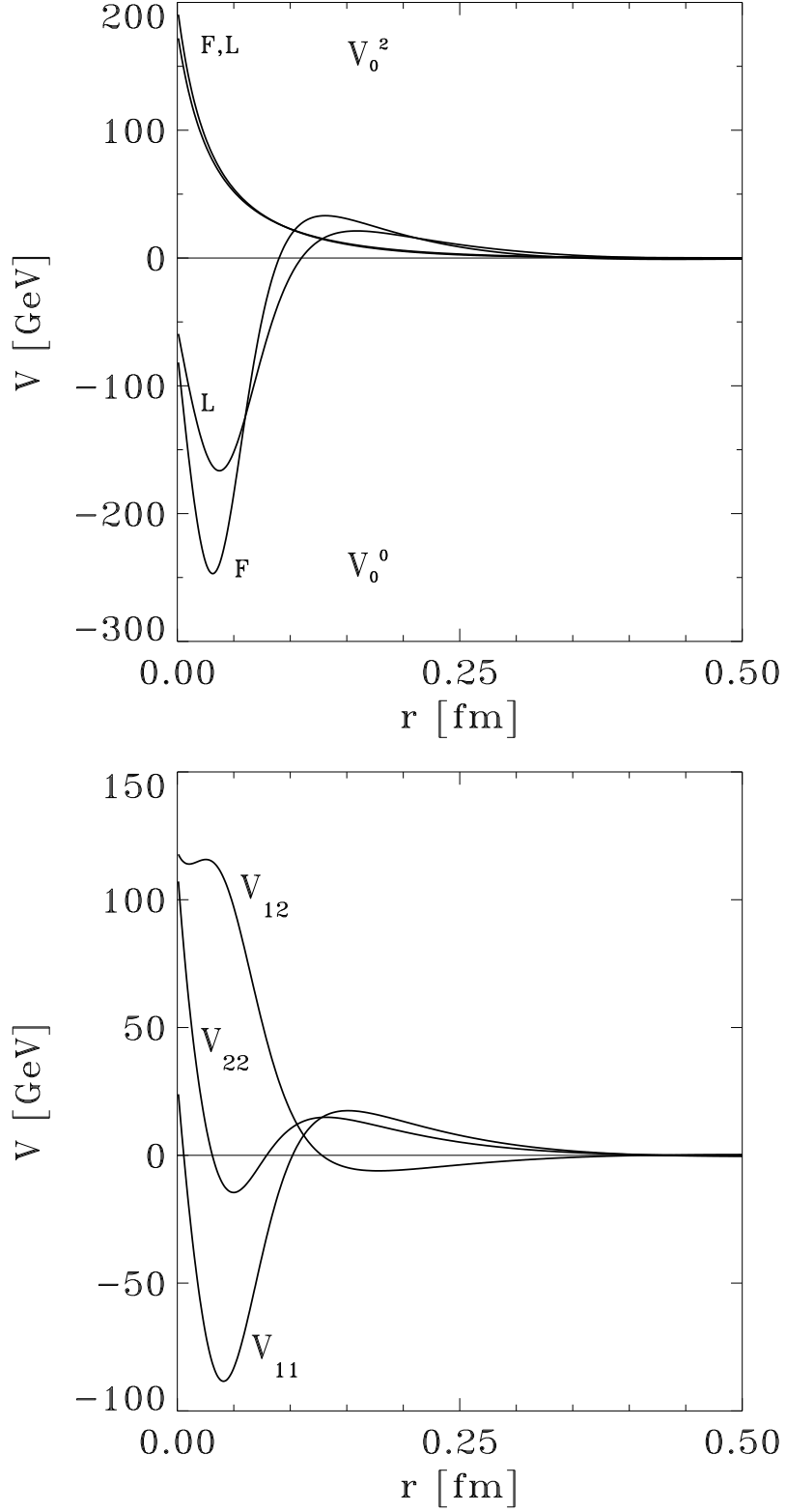


FIG. 2. Inversion potentials based on Froggatt [1] and Lohse [2] phase shifts (top) and derived potential matrix from inversion of Lohse data [2] (bottom).

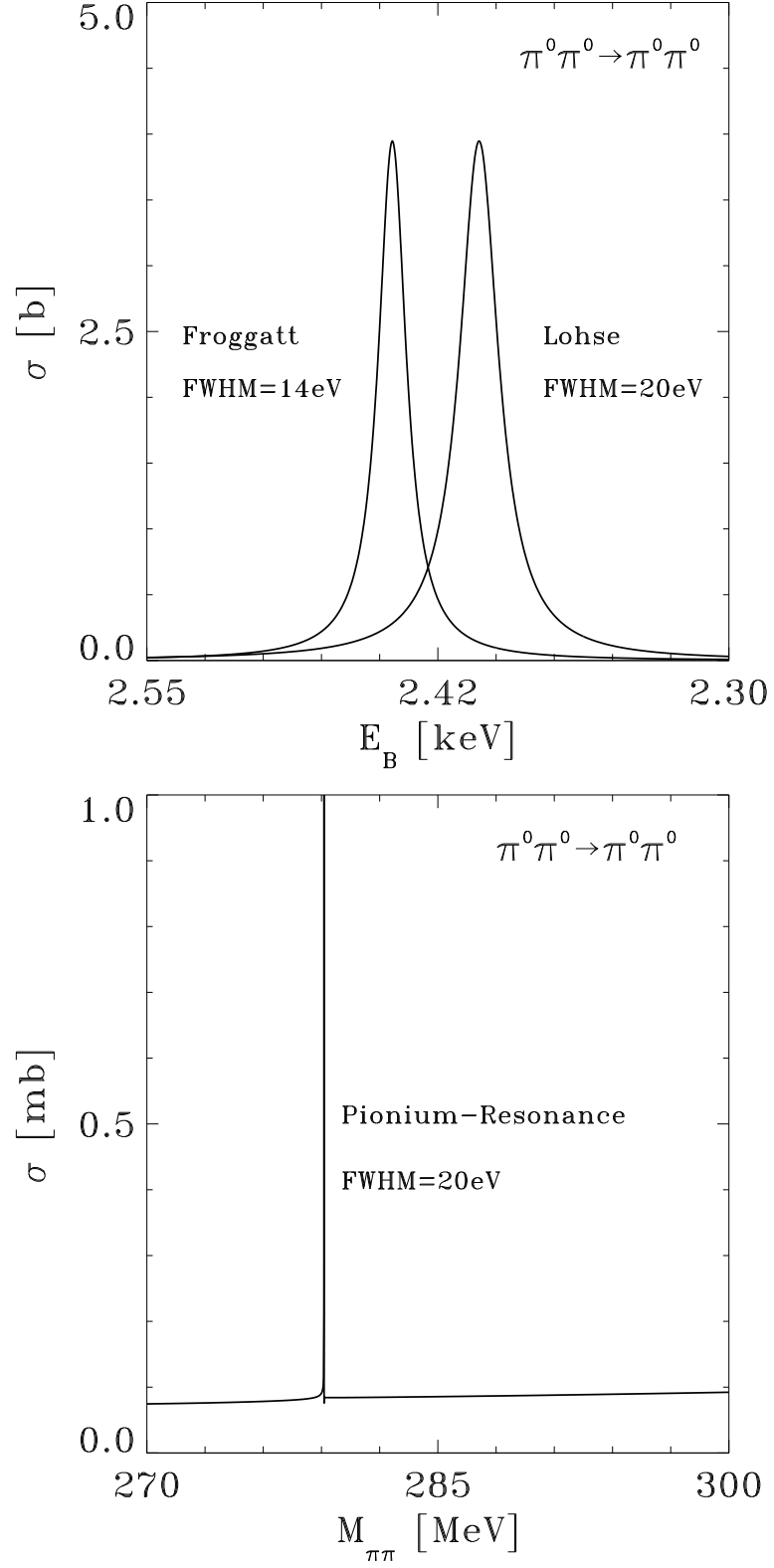


FIG. 3. High resolution pionium proper resonance (top) and general elastic cross section with $n = 1$ resonance (bottom).

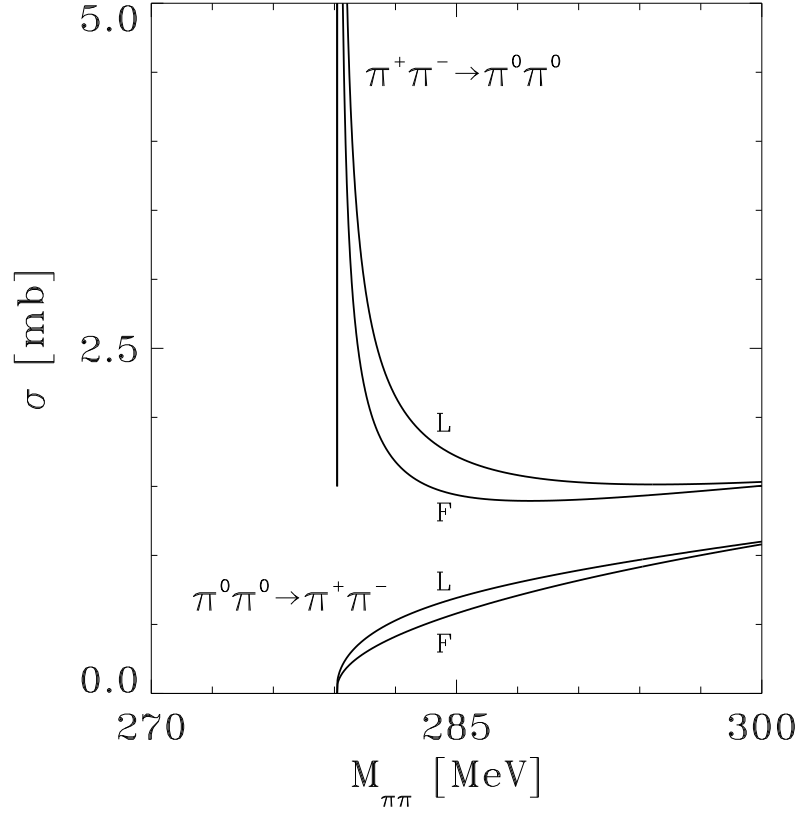


FIG. 4. Transition cross sections based on Froggatt [1] and Lohse [2] phase shift inversion.